CP violating phases in $\mu - e$ conversion

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Abstract

Experiments are planned to improve the sensitivity of $\mu - e$ conversion from the current $\sim 10^{-12}$ to $\sim 10^{-16} - 10^{-18}$. If the muon (bound to the nucleus) could be polarised, a spin asymmetry of the final state electron is sensitive to CP violating phases on lepton flavour violating operators. This is similar to extracting phases from asymmetries in the final state spin and phase space distributions of $\mu \to 3e$ and $\mu \to e\gamma$.

1 Introduction

CP is a discrete transformation, turning particules into anti-particles. In the Lagangian, it can be implemented by taking the complex conjugate of all the coupling constants. So CP Violation(CPV) can arise when the coupling constants have unremoveable phases. In the quark sector of the SM, there is one such unremoveable phase, and it is observed to be of order one. However, if there is new physics at the weak scale, the non-observation of electric dipole moments implies that combinations of the new physics phases must be small. The origin of CP Violation thus remains an enigma—are all coupling constants equiped with $\mathcal{O}(1)$ phases, or is CPV a particularity of the quark sector?

A source of CPV, beyond the phase of the quark mixing matrix, seems required to generate the excess of matter over anti-matter observed in the Universe. Some new physics is also required to generate neutrino masses, so the seesaw mechanism [1] is a popular extension of the SM because it naturally generates the observed small neutrino masses and can produce the baryon asymmetry via thermal leptogenesis [2, 3]. For this to occur, CP Violating phases are required in the lepton sector.

One of the major attractions of proposed third generation neutrino beam facilities [4] (neutrino factory, β-beam or superbeam), is their sensitivity to the phase δ of the lepton mixing matrix. Unfortunately these are expensive machines. So it is interesting to enquire if leptonic CP Violation can be found somewhere else.

This letter considers the sensitivity of $\mu - e$ conversion [5] to the phases of dipole interactions (see eqn (2)). This would be ambiguous evidence for CP Violation in the lepton sector, because Lepton Flavour Violating (LFV) processes like $\mu - e$ conversion require new physics at the electroweak scale, which may not be the same new physics as generates neutrino masses. For instance, in the supersymmetric seesaw, the CPV phases appearing in $\mu - e$ conversion could arise from the neutrino Yukawa couplings via renormalisation group running, or could be intrinsic to the soft supersymmetry-breaking parameters. Nonetheless, CPV is elusive, so any observation is interesting.

The asymmetries we are interested in arise from "triple products" of spin and/or momentum three vectors [6], and are sometimes refered to as T-odd asymmetries. Technically, they arise when the i from the Dirac trace $\gamma^\alpha \gamma^\beta \gamma^\sigma \gamma^\delta \gamma^5 = -4i\varepsilon^{\alpha\beta\sigma\delta}$, multiplies a CPV phase from the coupling constants (the case we are interested in), or the Imaginary part of an amplitude (*eg* induced by on-shell intermediate particles in a loop). Such a "triple product" asymmetry could manifest itself, for instance, as a forward-backward asymmetry in decays of a particle. Notice that it does not require the simultaneous presence of an phase in the amplitude and the coupling constants, as is required to obtain a difference between the integrated (over final state momentum and spin) decay rates of the particle and antiparticle [1].

Asymmetries which are sensitive to CPV phases, that can arise in $\mu \to 3e$, are reviewed in [5] and discussed in [7]. Farzan [8] showed recently that CPV phases could be measured in $\mu \to e\gamma$, if the γ and e polarisation

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[1] $S-$matrix unitarity relates $\langle(\vec{p}_1, \vec{s}_1)...(\vec{p}_n, \vec{s}_n)|(\vec{k}_1, \vec{s}_1')...(\vec{k}_m, \vec{s}_m')\rangle$ to $\langle(\vec{k}_1, \vec{s}_1')...(\vec{k}_m, \vec{s}_m')|(\vec{p}_1, \vec{s}_1)...(\vec{p}_n, \vec{s}_n)\rangle$. T transforms $\langle(\vec{p}_1, \vec{s}_1)...(\vec{p}_n, \vec{s}_n)|(\vec{k}_1, \vec{s}_1')...(\vec{k}_m, \vec{s}_m')\rangle \to \langle(-\vec{k}_1, -\vec{s}_1')...(-\vec{k}_m, -\vec{s}_m')|(-\vec{p}_1, -\vec{s}_1)...(-\vec{p}_n, -\vec{s}_n)\rangle$.

could be measured. Here we focus on μe conversion. The current bounds on Titanium ($Z = 22$) and gold ($Z = 79$) from Sindrum2 at PSI [9] are

$$\frac{\Gamma(\mu \mathrm{Ti} \to e\mathrm{Ti})}{\Gamma(\mu \mathrm{Ti} \to capture)} < 4.3 \times 10^{-12} \quad , \qquad \frac{\Gamma(\mu \mathrm{Au} \to e\mathrm{Au})}{\Gamma(\mu \mathrm{Au} \to capture)} < 7 \times 10^{-13} \quad . \tag{1}$$

Recall that the branching ratio scales $\propto Z$[5]. There are experiments under discussion ($\mu 2e$ at FermiLab[10], PRISM/PRIME [11] at J-PARC) aiming for sensitivities of $10^{-16} - 10^{-18}$. This letter assumes that the muon can be polarised, which is not automatic; the possibility of polarising the muon is discussed in [12].

2 $\mu - e$ conversion

Consider the effective dipole interaction, between a μ, an e and a photon, described by:

$$-\frac{4G_F m_\mu}{\sqrt{2}} \overline{\mu} \sigma_{\mu\nu}(A_L P_R + A_R P_L) e F^{\mu\nu} + h.c \tag{2}$$

where $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$, and the potentially complex dimensionless coefficients A_L and A_R arise by integrating out new physics in the vertex correction [13, 14, 15, 16]. In addition of eqn (2), New Physics can induce monopole ($\propto q^2 \overline{\mu} \not{A}\, e$) and four-fermion interactions, which are neglected here. In some (for instance, supersymmetric) models, the dipole interactions are the most significant. The interactions of eqn (2) allow the decay of a μ of momentum and spin p, s_μ to an e of momentum and spin k, s_e, and a γ of momentum q. The total branching ratio, integrated over phase space and summed on spins, is insensitive to the phases on A_L and A_R:

$$BR(\mu \to e\gamma) = 384\pi^2(|A_L|^2 + |A_R|^2) \tag{3}$$

To find triple products which are sensitive to CPV phases, consider instead the differential rate, or better the unintegrated |matrix element|^2, not summed over spins. It must be Lorentz invariant. If one can construct a Lorentz invariant CP-odd combination of spins and momenta, then one could enquire if it multiplies the potentially CP-odd combination $A_L A_R^*$. If yes, then there would be a CP asymmetry in the differential rate. CP odd, Lorentz invariant combinations of 4-vectors could be

$$\epsilon_{\alpha\beta\rho\sigma} p_\mu^\alpha k^\beta s_\mu^\rho s_e^\sigma \quad , \quad \epsilon_{\alpha\beta\rho\sigma} p_\mu^\alpha q^\beta s_\mu^\rho s_e^\sigma \tag{4}$$

These can appear in the Dirac traces, in the presense of a γ_5. In the |matrix element|^2, appear always two powers of the photon four-momentum q, so the first possibility is multiplied by $q^2 = 0$. The second possibility does not appear. It was recently shown by Farzan [8], that one must also include the spin of the photon, if one wishes to find a CP asymmetry in the $\mu \to e\gamma$ decay. Alternatively, one can study the decay $\mu \to 3e$, which has $q^2 \neq 0$, where there is an asymmetry sensitive to the relative phase between A_L and A_R[8] [2] . Also in $\mu - e$ conversion, $q^2 \simeq m_\mu^2$ [17].

In $\mu - e$ conversion experiments, an incident μ^- is captured by a nucleus, then cascades down to the $1s$ state. The muon usually turns into a neutrino, by muon capture β decay, or it could convert to an electron in the presence of New Physics. A detailed discussion of how to compute $\mu - e$ conversion rates, from effective interaction including eqn (2), can be found in [17, 18]. In the approximation that (2) is the only interaction permitting $\mu - e$ flavour change, the transition takes place in the electric field of the nucleus, with a matrix element [18]

$$\mathcal{M} = \mathcal{M}_L + \mathcal{M}_R = -\frac{4G_F m_\mu}{\sqrt{2}} \int d^3x e^{-i\vec{k}\cdot\vec{x}} \left(A_L^* \overline{u}(k) 2\sigma^{0i} E_i P_L \psi_{1s}^{(\mu)} + A_R^* \overline{u}(k) 2\sigma^{0i} E_i P_R \psi_{1s}^{(\mu)} \right) \tag{5}$$

where $\vec{E}$ is the electric field of the nucleus, $\psi_{1s}^{(\mu)}$ is the wavefunction of the muon in the $1s$ state, and the electron has been approximated (as in [19]) as a free plane wave of momentum $\vec{k}$. This approximation is less good for heavy nuclei [17, 18].

Define the average over the muon wavefunction:

$$\int d^3x \psi_{1s}^{(\mu)} E^i \equiv \langle E^i \rangle \quad ,$$

[2]Other triple product asymmetries in $\mu \to 3e$ are sensitive to other New Physics phases[15]

then using $2\langle E\rangle_i \sigma^{0i} = i\gamma^0 \langle \not{E}\,\rangle$, defining $\langle \not{E}\,\rangle^* = (\langle E\rangle^i)^* \gamma_i$ and neglecting m_e, one obtains

$$\begin{aligned}
\frac{|\mathcal{M}|^2}{8G_F^2 m_\mu^2} &= \mathcal{M}_L\mathcal{M}_L^* + \mathcal{M}_L^*\mathcal{M}_R + \mathcal{M}_L\mathcal{M}_R^* + \mathcal{M}_R\mathcal{M}_R^* \\
&= |A_L|^2\Big\{\not{k}\;\; \gamma^0\langle \not{E}\,\rangle P_L\gamma^0 P_R\gamma^0\langle \not{E}\,\rangle^*\Big\} + |A_R|^2\Big\{\not{k}\;\; \gamma^0\langle \not{E}\,\rangle P_R\gamma^0 P_L\gamma^0\langle \not{E}\,\rangle^*\Big\} \\
&\quad + A_L A_R^*\Big\{\not{k}\;\; \gamma^0\langle \not{E}\,\rangle P_R\gamma^0 P_R\gamma^0\langle \not{E}\,\rangle^*\Big\} + A_L^* A_R\Big\{\not{k}\;\; \gamma^0\langle \not{E}\,\rangle P_L\gamma^0 P_L\gamma^0\langle \not{E}\,\rangle^*\Big\} \qquad (6)
\end{aligned}$$

where inside curly brackets one should take a Dirac trace. The $A_L A_R^*$ cross terms drop out because of the chirality projection operators, and with $k_0 = m_\mu$, this gives

$$\Gamma = 64\pi G_F^2 m_\mu^3 |\langle E\rangle|^2 (|A_R|^2 + |A_L|^2) \qquad (7)$$

The conversion rate for polarised muons and electrons, can be obtained using the polarisation projection operators:

$$\frac{1}{2}\left(I + \gamma_5 \not{s}\;\right) \qquad (8)$$

(where s_α is the spin of particle α) to the wavefunctions. The $A_L A_R^*$ part of $|\mathcal{M}|^2$ is

$$\begin{aligned}
\mathcal{M}_L\mathcal{M}_R^* + h.c. &= -2G_F^2 m_\mu^2 A_L A_R^* \Big\{(I + \gamma_5 \not{s}_e)\not{k}\;\; \gamma^0\langle \not{E}\;\; P_R(I + \gamma_5 \not{s}_\mu)\rangle \gamma^0 P_R\; \gamma^0 \langle \not{E}\,\rangle^*\Big\} \\
&\quad -2G_F^2 m_\mu^2 A_L^* A_R \Big\{(I + \gamma_5 \not{s}_e)\not{k}\;\; \gamma^0\langle \not{E}\;\; P_L(I + \gamma_5 \not{s}_\mu)\rangle \gamma^0\; P_L \gamma^0 \langle \not{E}\,\rangle^*\Big\} \\
&= 8G_F^2 m_\mu^2 \mathrm{Im}\{A_L A_R^*\} |\langle \vec{E}\rangle|^2 \vec{s}_\mu \cdot (\vec{s}_e \times \vec{k}) \qquad (9)
\end{aligned}$$

where $\langle \vec{s}_\mu \cdot \vec{E}\rangle = 0$, assuming a radial $\vec{E}$ field and all the muons polarised.

To measure this process, suppose that the muon can be polarised by polarising the target, as suggested in [12]. Consider a coordinate system where $y - z$ is the plane of the outgoing electron and the muon polarisation:

$$\hat{z} = \hat{s}_\mu \qquad \vec{k} = |\vec{k}| \sin\theta\; \hat{y} + |\vec{k}| \cos\theta\; \hat{z} \qquad (10)$$

Then one wishes to measure electron polarisation in the perpendicular direction $\hat{x}$, which means in a cylinder aligned on the polarisation direction of the muon.

For 100% polarised muons, the asymmetry between electrons polarised in the $+\hat{x}$ and $-\hat{x}$ direction, normalised by the rate eqn (7), is :

$$\frac{2\int d\Omega \vec{s}_\mu \cdot (\vec{s}_e \times \hat{k})}{4\pi} \frac{\mathrm{Im}[A_L^* A_R]}{2(|A_L|^2 + |A_R|^2)} \leq \frac{\mathrm{Im}[A_L^* A_R]}{8(|A_L|^2 + |A_R|^2)} \qquad (11)$$

which would be maximised for extensions of the SM that give similar, and complex, contributions to A_L and A_R. The relative magnitude of A_L and A_R can be determined by the angular distribution of the electron, in $\mu \to e\gamma$, or $\mu - e$ conversion providing the muon is polarised.

In summary, , Lepton Flavour Violating rates are an important ingredient in determining the New Physics at the TeV scale. Experiments are planned to improve the sensitivity of $\mu - e$ conversion on nuclei, as compared to the rate for nuclear β decay by muon capture, to $10^{-16} - 10^{-18}$. If the muon could be polarised (and the electron polarisation is measured), a spin asymmetry in $\mu - e$ conversion is sensitive to CP violating phases. This is shown in this letter for the case where New Physics induces significant dipole interactions (see eqn (2)), but other dimension six interactions are negligeable. We plan a more complete analysis [20].

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